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3. REPORT TYPE AND DATES COVERED

2

4. TITLE AND SUBTITLE

Mathematical Analysis of Transient Reactive Gasdynamics Phenomena

5. FUNDING NUMBERS

DAAL03-88-K-0111

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DTIC ELECTE JAN 13 1992 S D

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

ARO 25753.9-MA

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*) Mathematical modelling and analysis are described for studies of the dynamical response of gaseous and solid materials (inert or reactive) to localized rapid power deposition. Theoretical, analytical and numerical methods are developed to find general properties of nonlinear equation systems and explicit solutions for specific initial-boundary value problems. Emphasis is placed on physical interpretation of mathematical results for the transient evolution of spatially distributed thermal and gasdynamic responses. New results are given for thermal explosion processes, for transient development of shocks and detonations, for the role of nonlinear acoustics in gascompression and for thermoacoustically induced mass transfer.

14. SUBJECT TERMS

thermal explosions, reactive gasdynamics, detonations, shocks, nonlinear acoustics, perturbation methods.

15. NUMBER OF PAGES

15

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

92-00317

92 1 6 103

MATHEMATICAL ANALYSES OF TRANSIENT REACTIVE GASDYNAMICS PHENOMENA

FINAL REPORT

D. R. KASSOY AND J. BEBERNES

SEPTEMBER 16, 1991

GRANT NUMBER: DAAL 03-88-K-0111

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I. Statement of Problem

The thematic research objective of our mathematical modeling and analysis activities during the past grant period has been to develop a quantitative understanding of the dynamical response of a gaseous or solid material to localized rapid power deposition. Essential time and length scales are obtained from a study of nondimensionalized equation systems. Perturbation methods are used to reduce complex mathematical systems to more elementary forms, valid in some specifically defined limit. Then analysis and numerical methods are used to find basic general properties of the equation systems and solutions for specific boundary and initial conditions. The transient evolution of thermal and gasdynamical responses to internal and external power deposition and mechanical disturbances is considered for both inert and reactive systems. The summary of activities to follow describes published, submitted and "in-preparation" papers in each of several research areas, and includes a brief comment about the objectives of the studies and the methods employed.

II. Summary of Results and Publication List

A. Dynamic Compression of a Gas in a Cylinder

The goal of this study is to describe the transient acoustic processes, including weakly nonlinear shock formation, occurring in a confined gas as it is compressed or expanded by a moving, low Mach number piston. Physical conditions correspond to those in an internal combustion engine. Fourier decomposition methods are used to describe both the linear and nonlinear phases of the gasdynamic events. Solutions are given in terms of asymptotic expansions, valid in the small piston Mach number limit.

1. Meng Wang and D. R. Kassoy, "Dynamic Response of an Inert Gas to Slow Piston Acceleration," J. Acoust. Soc. Am. **87** (1990), 1466–1471.

   Abstract. This study describes a theoretical investigation of weak acoustic wave fields generated in a confined inert gas during slow piston acceleration processes. The piston velocity increases continuously from a zero initial value to a characteristic value, on the time scale of the piston passage from bottom to top of a cylinder. In contrast, the initial piston acceleration is finite. The gasdynamic phenomena occurring in the system are modeled by the unsteady, planar Euler equations, which are solved asymptotically in terms of the small characteristic piston Mach number ϵ. The finite initial piston acceleration is shown to initiate $O(\epsilon^2)$ acoustic disturbances in the gas. A combination of multiple-time-

scale methods and Fourier series techniques are used to develop simultaneous descriptions of the evolving acoustic field and the accumulated bulk response of the gas to piston motion. It is found that the wave field remains linear throughout the period of piston motion. The wave amplitude is proportional to the initial acceleration of the piston and increases with the mean density of the gas.

2. Meng Wang and D. R. Kassoy, "Dynamic Compression and Weak Shock Formation in an Inert Gas Due to Fast Piston Acceleration," J. Fluid Mech. **220** (1990), 267–292.

   Abstract. Unsteady gasdynamic concepts are used to model the piston-driven compression of a confined gas. Perturbation methods, based on the limit of small piston Mach number, are used to construct solutions. The piston Mach number increases smoothly from zero to a maximum value, $M_p = O(10^{-2})$ during an acoustic time period $t_a^* = O(10^{-4}\ s)$. A linear acoustic field is generated and is represented in terms of an infinite series of Fourier spatial modes. During the longer piston time period $t_p^* = O(10^{-2}\ s)$ the piston moves at constant speed. A multiple-timescale formulation is used to separate the instantaneous acoustic field from the accumulated bulk response of the gas to piston compression. The latter is found to be identical to the classical quasistatic results from equilibrium thermodynamic calculations. Nonlinear effects become important on the piston timescale. Modal interactions are represented by a system of coupled, nonlinear ordinary differential equations for the time-dependent Fourier coefficients. A numerical solution for this system describes the wavefront steepening to form a weak shock and its propagation back and forth repeatedly inside the cylinder.

3. Meng Wang and D. R. Kassoy, "Evolution of Weakly Nonlinear Waves in a Cylinder with a Movable Piston," J. Fluid Mech. **221** (1990), 23–52.

   Abstract. Small-amplitude wave motion in an inert gas confined between a moving piston and a fixed cylinder endwall is studied using the unsteady Euler equations. The waves generated by either initial disturbances or piston motion, reflect back and forth in the cylinder on the acoustic timescale. The accumulated effect of these waves controls the bulk variations of velocity and thermodynamic variables on the longer piston timescale. Perturbation methods, based on the small ratio of acoustic to piston time, are employed to formulate the gasdynamic problem. The application of multiple timescaling allows the gasdynamic wave field to be separated from the bulk response of the gas. The evolution of the wave phenomena, including nonlinear wave deformation and weak shock formation during the piston passage time, is described in terms of time-dependent Fourier series

solutions, whose coefficients are computed from a truncated system of coupled nonlinear ordinary differential equations. The long-time asymptotic flow field after shock formation is sawtooth-like, in which case the Fourier modes become decoupled. A remarkably simple relation between the shock amplitude and piston velocity is discovered. It is demonstrated that (i) the wave amplitude and frequency strongly depend on the piston motion; (ii) shock waves can be damped in a significant way by internal dissipation; and (iii) the mathematical approach developed in this study possesses certain advantages over the more traditional method of characteristics.

B. Shock Wave Initiation and Evolution

A boundary power deposition disturbance is used to create shock waves in an inert gas. An initial-boundary value analysis is used for both planar and spherical boundary geometries. Perturbation methods are used to find crucial time and length scales that are of importance in each of several physical regimes. Analytical and numerical methods are used to find solutions that describe transient shock wave and subsequent flow field evolution. Results are given for both constant and time-dependent boundary heating.

1. A. A. Sileem and D. R. Kassoy, "Shock Wave Generation and Propagation in Gases Due to Boundary Power Deposition," submitted (1991).

Abstract. The initiation, and subsequent propagation of a planar unsteady shock wave in an inert gas confined between two parallel plane walls is considered, when there is time-dependent power deposition at the boundary. A numerical solution is obtained for the Euler equations that describe the flow driven by the moving edge of an expanding conduction layer as formulated in earlier work by Kassoy and co-workers. The solution is obtained by using the explicit, finite-difference MacCormack scheme with the Flux-Corrected Transport technique to reduce the oscillations wherever large gradients exist.

The results show that the numerical code is capable of capturing shock waves with excellent accuracy. The pressure and velocity at the edge of the conduction layer increase as the power deposition at the boundary increases with time. Accordingly, the shock speed and strength increase with time. The increase in the pressure level with time provides a source of compression heating in the gas contained in the conduction layer. The spatially variable fluid properties between the edge of the conduction layer and the shock are found for several different time-dependent functions of the power deposition at the boundary.

2. Sutrisno and D. R. Kassoy, "Weak Shocks Initiated by Power Deposition on a Spherical Source Boundary," SIAM J. Appl. Math. **51** (1991), 658–672.

Abstract. An inert compressible gas external to a spherical surface is in an equilibrium state initially. Heat is added at the sphere surface during a period that is short compared to the acoustic time of the sphere t'_a (the radius of the sphere divided by the equilibrium sound speed), but larger than the mean time between molecular collisions. Temperature increase in the conduction layer adjacent to the surface induces gas motion arising from thermal expansion. The motion at the layer edge has a piston-like effect on the gas beyond, generating a thicker layer in which a linear acoustic disturbance is found. As time elapses, an accumulation of nonlinear effects leads to the development of a weak shock in the acoustic region. The subsequent nonlinear evolution of the weak shock then depends mainly on the boundary heat flux. For relatively fast boundary heating, the nonlinear evolution process is described by a planar equation. On the other hand, during relatively slow boundary power addition, disturbances are able to propagate farther from the surface before nonlinear effects become important. The nonlinear evolution is then described by a spherical nonlinear equation. Solutions are sought in terms of asymptotic expansions valid when $t'_a/t'_c \to 0$, *where* t'_c *the conduction time of the sphere*, is the ratio of the square of the radius to the thermal diffusivity of the gas in the initial state.

C. Detonation Initiation and Evolution

These studies have been focused on the birth and evolution of planar detonations arising from purely thermal disturbances in a reactive gas. An initial-boundary value problem approach has been used where the initiating disturbance is either power deposition through a boundary, or as bulk internal heating. Both Navier–Stokes and Euler equation numerical codes have been written and employed to find explicit solutions. Numerical data has been evaluated to demonstrate the propagation of gasdynamic and reactive waves, the appearance of localized thermal explosions and the formation of ZND-type detonations.

1. J. F. Clarke, D. R. Kassoy, N. E. Meharzi, N. Riley and R. Vasantha, "On the Evolution of Plane Detonations," Proc. R. Soc. Lond. A, **429** (1990), 259–283.

Abstract. Numerical solutions of the Navier–Stokes equations for the plane one-dimensional unsteady motion of a compressible, combustible gas mixture are used to follow the history of events that are initiated by addition of large heat power through a solid surface bounding

an effectively semi-infinite domain occupied by the gas. Plane Zel'dovitch–von Neumann–Doring detonations eventually appear either at the precursor shock (which exists in every set of circumstances) or in the regions, occupied by an unsteady induction-domain and an initially quasi-steady fast-flame, that lie behind the precursor shock.

2. A. A. Sileem, D. R. Kassoy and A. R. Hayashi, "Thermally Initiated Detonation Through Deflagration to Detonation Transition," Proc. R. Soc. Lond. A, in press, (1991).

Abstract. The initiation of a planar detonation via deflagration to detonation transition (DDT) is studied in a reactive mixture confined between two infinite parallel plane walls. The mixture is ignited by bulk power deposition of limited duration in a thin layer adjacent to the left-hand wall. A combustion wave starts to propagate into the reactant, supported by expansion of the burned hot gases. Compression waves generated ahead of the combustion front coalesce quickly to form a shock wave strong enough to trigger considerable chemical reaction. This newly started reaction evolves into a reaction center in which the chemical heat release rate increases rapidly. The subsequent explosion of the reaction center creates compression waves that steepen to form a new shock. The strengthened lead shock ignites a new strongly coupled reaction zone that supports the formation of an initially overdriven detonation. Subsequently, the wave decays to an oscillating planar detonation with mean properties of a C. J. wave.

D. Steady State High Speed Reaction Zone Structure

This research effort is focused on describing the flow, thermal and chemical structure in the reaction zone behind a steady state normal shock. A major goal is to predict the length scales for the reactive processes, given specific chemical kinetic steps. Perturbation methods, developed originally in ARO-supported research on classical thermal explosion theory, are used to develop solutions for this compressible flow problem.

1. K. Kirkkopru and D. R. Kassoy, "Induction Zone Structure for a High-Speed Deflagration with Variable Mole Chemistry," Phys. Fluids A 1 (1989), 874–880.

Abstract. The induction zone characteristics of a planar subsonic high-speed reactive flow downstream of a specific origin are investigated theoretically for the global irreversible reaction $F + Ox \rightarrow \nu P$. The equation of state for the reacting gas mixture is more general than that for a constant molecular weight gas. Perturbation methods based on the limit of high activation energy are used to construct the general parameter-dependent analytical

solutions. The dependence of the ignition delay distance on the kinetic, stoichiometric, and flow parameters is discussed in detail. Significantly, it is shown that the reaction with a mole decrement ($\nu = 1$) yields the minimum, and a mole increment ($\nu = 3$) yields the maximum ignition delay distance when the chemical heat addition and the origin values of parameters are fixed. The physics and length scales found from the perturbation analysis are used as a guide in generating supporting numerical solutions.

2. K. Kirkkopru and D. R. Kassoy, "High Speed Reaction Zone Structure for Variable Mole Chemistry," SIAM J. Appl. Math. **51** (1991), 1090–1118.

   Abstract. The structure of a high-speed deflagration downstream of a specific origin is investigated theoretically for the exothermic irreversible Arrhenius-type reaction $F + Ox \rightarrow \nu P$. Perturbation methods, and in particular a nonlinear transformation used in thermal explosion theory, are used to find fully analytical solutions for the spatial structure of the high speed reaction zone when the activation energy is large. In the rapid reaction region beyond the induction zone, there is a strong interaction between large chemical heat release and flow compressibility. When the chemical reaction causes a mole reduction ($\nu < 2$) and there is sufficiently large heat release, the flow velocity reaches a *maximum and then* declines while the temperature increases monotonically throughout the process. Significantly, it is shown that the flow cannot evolve to the Chapman–Jouguet (C.J.) state where the final local Mach number is unity and the reactant concentration is zero. When the mole number of the evolving flow is constant or increasing ($\nu \geq 2$), the flow velocity increases monotonically and a final C.J. state can develop when the right amount of chemical heat addition is available for a given input Mach number at the origin. The asymptotic solutions compare quite favorably with those found numerically from an integration of the transport-free compressible, reactive flow equations.

3. K. Kirkkopru and D. R. Kassoy, "Dissociation–Recombination Relaxation Behind a Normal Shock," in press, Phys. Fluids (1991).

   Abstract. Mathematical methods developed originally for thermal explosion theory are used to study the evolution of a dissociation–recombination process occurring in a compressible gas behind a normal shock. The dissociation–recombination reaction, $AB + M \rightleftharpoons A + B + M$, is assumed to have a relatively large dissociation temperature, so that high activation energy perturbation techniques can be used to derive general parameter dependent *analytical* solutions. Spatial variation of the dependent variables is described in three zones, each of distinct length scale and physical character. Initially,

small changes in temperature, concentrations, density and pressure occur in the relatively high temperature dissociation initiation zone. In the subsequent broader major dissociation region, most of the specie AB is converted to A and B and there are significant variations in all physical variables. In the last and thickest zone, recombination becomes important as the reactive flow evolves to a final equilibrium state. The results provide an analytical counterpart to numerical solutions obtained earlier for the now classical problem of idealized diatomic gas dissociation.

E. Mass Transport Due to Internal Power Deposition

The objective of this work is to predict quantitatively the mass transport induced when spatially distributed, transient power deposition occurs in a confined inert gas. Perturbation methods and asymptotic expansions, valid for the low Mach number (of the induced gas velocity) limit, are used to develop solutions.

1. A. Herczynski and D. R. Kassoy, "Response of a Confined Gas to Volumetric Heating in the Absence of Gravity I. Slow Transients," Phys. Fluids A **3** (1991), 566–577.

   Abstract A one-dimensional model for bulk motion induced by a transient volumetric heat source in a confined gas at zero-gravity is considered. Rational approximation methods are used to derive a quantitative theory for the gas response to a spatially distributed, time dependent internal power deposition. The resulting low Mach number compressible flow equations are solved by using perturbation methods. Solutions are given for a conduction free core and thin conductive boundary layers adjacent to the endwalls. It is found that any spatially nonuniform power deposition will cause fluid motion. Net mass transport in the closed container will occur for certain spatially distributed heating. The model mimics the thermal effects of an exothermic gas phase reaction in vapor transport experiments conducted in space. The solutions demonstrate that thermally induced mass transport can be as large as diffusive mass transport in a typical experiment.

2. Sutrisno and D. R. Kassoy, "The Interaction of Thermoacoustic and Natural Convection in a Confined Gas Subjected to Boundary Heating," submitted (1991).

   Abstract. Thermal power deposition through the vertical surfaces of a rectangular container enclosing gaseous helium initiates fluid motion through thermoacoustic and buoyancy effects. When the surface temperature rise on the acoustic timescale of the container is comparable with the initial value, conduction heating takes place only in a thin, con-

tinuously growing boundary layer near the heated wall. The horizontal expansion of the hot gas produces a piston-like effect at the boundary layer edge. The edge of the expanding conduction layer induces compressive acoustic disturbances in the essentially isentropic core region. Repeated passage of the acoustic disturbances across the core produces constantly increasing, accumulated temperature, pressure and density disturbances. A vertically aligned downward gravity field creates buoyancy forces in the hot conductive boundary layers that cause upward natural convection of the heated fluid. On the other hand, the fluid motion in the unheated, and acoustically compressed core, is downward, the result of a balance between weak buoyant forces, and an induced vertical pressure gradient. A systematic analysis using various perturbation techniques is carried out to obtain solutions in both the boundary layer and the core. The theory is based on a complete system of conservation equations governing viscous, heat conducting and compressible flow.

F. Thermal Explosion Phenomena

The object of this study is to give a precise, accurate description of the ignition process for reactive–diffusive and reactive-Euler induction period models. For reactive–diffusive self-ignition processes, we can now mathematically describe the progress of self-ignition right up to the time when combustion ceases in the core of the hot spot, very little hindered by diffusive losses.

1. J. Bebernes, A. Bressan, D. Kassoy, and N. Riley, *The confined nondiffusive thermal explosion with spatially homogeneous pressure variation*, Combustion Science and Technology, **63** (1989), 43–62.

   Abstract. A solution is developed for a nondiffusive thermal explosion in a reactive gas confined to a bounded container Ω with a characteristic length ℓ'. The process evolves with a spatially homogeneous time-dependent pressure field because the characteristic reaction time t'_R is large compared to the acoustic time ℓ'/C'_0 where C'_0 is the initial sound speed. Exact solutions, in terms of a numerical quadrature are obtained for the induction period temperature, density, and pressure perturbations as well as for the induced velocity field. Traditional single-point thermal runaway singularities are found for temperature and density when the initial temperature disturbance has a single-point maximum. In contrast, if the initial maximum is spread over a finite subdomain of Ω, then the thermal runaway occurs everywhere. Asymptotic expansions of the exact solutions are used to provide a

complete understanding of the singularities. The perturbation temperature and density singularities have the familiar logarithmic form $-\ln(t_e' - t')$ as the explosion time t_e is approached. The spatially homogeneous pressure is bounded for single-point explosions but is logarithmically singular when global runaway occurs. Compression heating associated with the unbounded perturbation pressure rise is the physical source of the global thermal runaway.

2. J. Bebernes and D. Kassoy, *Reactive-Euler induction models*, Transactions of the Sixth Army Conference on Applied Mathematics and Computing, ARO Report 89-1, 473–482.

   Abstract. A unified formulation for the induction period for thermal reaction problems is presented using high activation energy asymptotics. The important parameters in the nondimensional equations are the rations of characteristic reaction, acoustic, and conduction times in the thermally disturbed parcel of a reactive gas of dimension L. In larger systems, transport effects are negligible and the induction period is controlled by reactive gasdynamics equations. Two of these models are analyzed.

3. J. Bebernes and A. Bressan, *Blowup for some reactive-Euler induction models*, Reaction–Diffusion Equations, (K. J. Brown and A. A. Lacey, eds.), Clarendon Press, Oxford, 1990, pp. 1-24.

   Abstract. A unified formulation for the induction period for thermal reaction problems is presented using high activation energy asymptotics. The important parameters in the nondimensional equations are the ratios of characteristic reaction, acoustic, and conduction times in the thermally disturbed parcel of a reactive gas of dimension L. Two of these models are analyzed. For the second reactive-Euler model, solutions become unbounded within finite time and generically the blowup occurs at a single point. As the explosion time is approached, a precise description of the asymptotic profile of a solution is obtained.

4. J. Bebernes and D. Eberly, *Characterization of blowup for a semilinear heat equation with a convection term*, Quart. J. Mech. Appl. Math., **42** (1989), 447–456.

   Abstract. The problem $u_t = \Delta u + e^u - |\nabla u|^2$, $(x,t) \in \Omega \times (0,T)$ with $u(x,0) = u_0(x) \geq 0$ on Ω, and $u(x,t) = 0$ for $(x,t) \in \partial\Omega \times (0,T)$ has a gradient term which has a dampening effect on the reaction term tending to retard blowup. Its special nature allows a precise determination of how and where blowup does occur.

5. J. Bebernes and A. Lacey, *Finite time blowup for a particular parabolic system*, SIAM J. Math. Anal., **211** (1990), 1415–1425.

   Abstract. The problem of blowup in finite time is considered for an initial-boundary

value problem for a two-dimensional parabolic system which models exothermic chemical reactions taking place within a porous medium assuming one diffusing reactant and the usual Frank–Kamenetski approximation to the classical Arrhenius rate law.

6. J. Bebernes and D. Eberly, *Mathematical Problems from Combustion Theory*, Applied Mathematical Sciences Series 83, Springer–Verlag, New York, 1989.

   Abstract. This book is a research monograph covering problems in combustion theory that J. Bebernes and co-workers have analyzed over the past 5 years. A complete development of combustion models from basic conservation principles is given. Recent results in radial symmetry of solutions to elliptic problems are summarized. The solid fuel ignition model on radial domains is analyzed extensively; in particular, the questions of when, where, and how blowup occurs are studied. The complete solid fuel system is also studied in some detail. Finally, gaseous fuel models are analyzed, including both diffusive and nondiffusive models, and including a complex parabolic system.

7. J. Bebernes and A. Lacy, *Finite time blowup for reactive–diffusive systems*, J. Differential Equations, **93** (1991).

   Abstract. This paper deals with initial-boundary value problems for parabolic systems of the form

$$
(1.1)\qquad \begin{cases} \begin{matrix} u_t - \Delta u = (1-v)f(u) \\ v_t - \Delta v = (1-v)f(u) \end{matrix}, & x \in \Omega \subset I\!R^n, \quad t > 0 \\ u(x,0) = u_0(x) \geq 0, \quad 0 \leq v(x,0) = v_0(x) \leq 1, & x \in \Omega \\ \dfrac{\partial u}{\partial n} + \mu u = 0, \quad \dfrac{\partial v}{\partial n} + \nu v = 0, \quad \mu, \nu \in [0, \infty] & x \in \partial\Omega, \quad t > 0, \end{cases}
$$

where $\Omega \subset I\!R^n$ is a bounded domain with smooth boundary $\partial\Omega$, $u_0(x)$ and $v_0(x)$ are continuous, bounded, nonnegative functions, and $f(s)$ is assumed to be C^2 with $f(s) > 0$, $f'(s) > 0$, $f''(s) > 0$, and $f(s) \gg s^2$ for $s \to \infty$. Additional assumptions will be imposed as needed in the next three sections.

Problem (1.1) arises as an approximating model for an exothermic chemical reaction taking place within a porous medium assuming one diffusing reactant and the Frank–Kamenetski approximation $f(u) = e^u$ for the classical Arrhenius rate law. In this case u and v are chosen to that physically the scaled temperature of the reaction process above ambient is u and the scaled concentration is $1 - v$.

Problem (1.1) ([1], [2]) has a unique nonnegative classical solution on $\pi_\sigma = \Omega \times [0, \sigma)$. By this, we mean a pair of nonnegative $C^{2,1}$ functions $(u(x,t),\ v(x,t))$ which satisfy (1.1)

in π_σ. For a given solution (u, v) of (1.1), define

$$T = \sup \left\{ \sigma > 0 : (u, v) \text{ is a bounded solution of (1.1) on } \pi_\sigma \right\} \tag{1.2}$$

The purpose of this paper is to determine in terms of u_0, v_0, μ, ν, f and Ω the cases $T = +\infty$ and $T < +\infty$. If $T = +\infty$, the solution exists for all time $t > 0$ and is global. If $T < +\infty$, then

$$\limsup_{t \to T} \left[\|u(t)\|_\infty + \|v(t)\|_\infty \right] = +\infty \tag{1.3}$$

since otherwise the solution could be extended beyond T. When (1.3) holds, with $T < \infty$, we say that the solution *blows up in finite time.*

8. J. Bebernes and S. Bricher, Final time blowup profiles for semilinear parabolic equations via center manifold theory, *SIAM J. Math. Anal.*, to appear.

Abstract. In this paper we consider the semilinear parabolic equation $u_t = \Delta u + f(u)$ in $\mathbb{R}^n \times (0, \infty)$ where $f(u) = e^u$ or $f(u) = u^p$, $p > 1$. For any initial data that is a positive, radially decreasing lower solution, and that causes the corresponding solution $u(x, t)$ to blow up at $(0, T) \in \mathbb{R}^n \times (0, \infty)$, we prove by using techniques from center manifold theory that the final time blowup profiles satisfy:

$$u(x, T) = -2 \ln |x| + \ln |\ln |x|| + \ln 8 + o(1) \qquad \text{for } f(u) = e^u,$$
$$u(x, T) = \left(\frac{8\beta^2 p |\ln |x||}{|x|^2} \right)^\beta (1 + o(1)) \qquad \text{for } f(u) = u^p$$

as $|x| \to 0$.

9. J. Bebernes, S. Bricher, and V. Galaktionov, *Asymptotics of blowup for weakly quasilinear parabolic problems*, J. Math. Analysis and Applications, submitted.

Abstract. The blowup asymptotics of classical solutions $u = u(|x|, t)$ of the Cauchy problem and the initial-boundary value problem for the weakly quasilinear heat equation

$$u_t = \nabla \cdot (k(u) \nabla u) + Q(u)$$

and analyzed, assuming that $k(u) = 1 + p(u)$, $Q(u) = e^u(1 + q(u))$ with $p, q \to 0$ as $u \to \infty$ and small along with their first two derivatives. We prove that many asymptotic properties for the semilinear equation $u_t = \Delta u + e^u$ remain valid after weak quasilinear perturbations of the elliptic operator.

III. Participants

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IV. Report of Inventions

None.